LECTURE 10
EFFICIENT AND EQUITABLE TAXATION

I. OPTIMAL TAXATION WHEN THE GOVERNMENT CAN TAX ALL GOODS

- Equivalent to a lump sum tax

II. OPTIMAL TAXATION WHEN SOME GOODS CANNOT BE TAXED

A. The Ramsey Rule

B. Reinterpretation of the Ramsey Rule

Example 1, Example 2

- compensated/uncompensated elasticities
- calculate excess burden

You may want to review the following problems:

Finals

Spring 1995, Question 4

Fall 1995, Question 4

Winter 1996, Question 4

Spring 1996, Question 6

Fall 1997, Question 5

I. Optimal Taxation When the Government Can Tax All Goods: A tax on all goods at the same rate is equivalent to a lump sum tax.

The goal of the government may be to finance a budget with the minimum excess burden possible, and without using lump sum taxes (which are politically unfeasible). Assume there are three goods in the economy -- food, clothing, and leisure. The reason we will use three goods, rather than two, is because in the next section we will only impose a tax on two of the three goods. The utility function is represented by:

$$U = U(F, C, L)$$

And the budget constraint is written down as:

$$w(T - L) = P_F F + P_C C$$

where T is the time endowment. It can be rewritten as the "full budget constraint"

$$wT = P_F F + P_C C + wL$$

Consider the following ad-valorem taxes: tax goods F, C and L at the same rate J. This tax raises the prices to $(1+\tau)P_F, (1+\tau)P_C, (1+\tau)w$.

It follows that the budget constraint is now:

$$wT = (1+\tau)P_F F + (1+\tau)P_C C + (1+\tau)wL$$

-- or --

$$\frac{wT}{(1+\tau)} = P_F F + P_C C + wL$$

The key point is that a tax on all commodities, including leisure, at the same tax rate, is equivalent to reducing the value of the time endowment. Since w and T are fixed, wT is fixed. Therefore, equivalent to a lump sum tax!

This leads to the obvious point that a tax on all commodities at the same tax rate is *equivalent to a lump sum tax.*

On the other hand, it may be impossible to tax one of the good (for example, leisure). Taxing the two other goods, F and C, at the same tax rate is usually <u>not</u> efficient.

II. Optimal Taxation When Some Goods Cannot Be Taxed
A. The Ramsey Rule

Assuming the third good (leisure in this example) cannot be taxed, how should the government set taxes on food and clothing? The objective function of the government is to minimize total excess burden (TEB), subject to raising sufficient revenue (TREV). The implication is that the marginal excess burden per marginal government revenue raised from each commodity must be equal.

Suppose $\frac{MEB_F(\tau_F)}{REV_F(\tau_F)} > \frac{MEB_C(\tau_C)}{REV_C(\tau_C)}$. Then the government can raise the same revenue, but with less TEB to society. In this case it should lower the tax on food, and raise it on clothing.

Consider special case where F and C are unrelated commodities: they are neither substitutes or complements for each other. In addition, assume the supply curve for food is perfectly elastic with a price of P_0.

Now impose a small per-unit tax, u_F, on food. Graphically the TEB is the triangle ABC:

***** SEE FIGURE 10-1 *****

Thus $TEB = \frac{1}{2} u_F \Delta F$, and revenue raised $TREV = u_F F_1$.

To minimize TEB, we need to know information on the MEB of the last dollar of revenue collected. Graphically, increase the tax from u_F to u_F+1 (that is, raising the per-unit tax rate by $1). The difference between the new, bigger triangle and the old, smaller triangle is the MEB.

Step 1: find the MEB from tax increase
Step 2: compute the associated increase in tax revenue
Step 3: divide MEB by marginal increase in tax revenue.

Step 1: $TEB = \frac{1}{2}(u_F + 1)\Delta F$ (note that) F only technically correct for infinitesimal change)

Before the $1 tax increase, $TEB = \frac{1}{2}(u_F)\Delta F$; the difference between the two triangles gives $MEB = \frac{1}{2}\Delta F$.

Graphically this area is approximately the difference between the excess burden associated with the tax rate u_F+1 and the original tax u_F. It ignores the small trapezoid in the figure, however.

Step 2: The marginal government revenue is the new revenue minus the old revenue, $\left[(u_F + 1)F_1 - u_F F_1\right] = F_1$.

Step 3: Dividing the MEB per additional dollar raised gives: $\frac{1}{2}\left(\frac{\Delta F}{F_1}\right)$

Similarly the MEB per dollar raised on clothing would be: $\frac{1}{2}\left(\frac{\Delta C}{C_1}\right)$

The Ramsey Rule: When only a subset of the goods in the economy can be taxed, in equilibrium the MEB per dollar raised must be equal across those goods:

$$\frac{\Delta F}{F_1} = \frac{\Delta C}{C_1}.$$

This does <u>not</u> say that MEB is equalized across goods. Instead, MEB per dollar raised is equalized in order to be efficient.

Also note that the change in a variable,) , divided by its total value is just the percentage change in that variable (recall the definition of an elasticity $\left(\frac{\Delta Q}{Q}\Big/\frac{\Delta P}{P}\right)$). The Ramsey rule implies that to minimize total excess burden, tax rates should be set so that the percentage reduction in the quantity demanded of each commodity is the same.

B. A reinterpretation of the Ramsey rule

Assume goods F and C are unrelated, so that a price change in one market does not affect the other market. Recall that the excess burden on F can be written as $\frac{1}{2}\eta_F P_F F \tau_F^2$ and the excess burden on C can be written as $\frac{1}{2}\eta_C P_C C \tau_C^2$. By adding these two expressions together, the total excess burden is:

$$EB = \frac{1}{2}\eta_F P_F F \tau_F^2 + \frac{1}{2}\eta_C P_C C \tau_C^2$$

The government's problem is to minimize, through its choice of tax rates J and J_C, the total excess burden subject to a revenue constraint, $P_F F J_F + P_C C J_C = R$.

We can set this up as a Lagrangian:

$$\min_{\tau_F,\tau_C} L = \frac{1}{2}\eta_F P_F F \tau_F^2 + \frac{1}{2}\eta_C P_C C \tau_C^2 + \lambda\left[R - P_F F \tau_F - P_C C \tau_C\right]$$

Then set $\frac{\partial L}{\partial \tau_F} = 0 \Rightarrow \eta_F P_F F \tau_F - \lambda P_F F = 0 \Rightarrow \lambda = \eta_F \tau_F$.

Similarly for tax rate on C, set $\frac{\partial L}{\partial \tau_C} = 0 \Rightarrow \lambda = \eta_C \tau_C$. Thus $\eta_F \tau_F = \eta_C \tau_C$.

By rearranging the terms, we obtain the "inverse elasticity rule":

$$\frac{\tau_F}{\tau_C} = \frac{\eta_C}{\eta_F}$$

As long as two goods are unrelated in consumption, the tax rates should be inversely proportional to their elasticities. The higher the compensated elasticity on C relative to F, the low the tax rate on C. The intuition is as follows: an efficient set of taxes should distort behavior as little as possible. The government therefore taxes relatively inelastic goods.

Example #1

As a policy maker, you are provided with the following information.

- The compensated elasticity of demand for clothing with respect to the price, denoted as $0^{clothing}$, is equal to -0.5.
- The uncompensated elasticity of demand for clothing with respect to the price, denoted as $,^{clothing}$, is equal to -1.0.
- On average, 1,000 units of clothing are sold per year at a price of $10 per unit.
- The supply curve for clothing is perfectly elastic.

a. Calculate the excess burden from a 25 % ad-valorem tax on clothing. How about a tax of 15 %? 30 %?

b. If you were instead told that the elasticity of supply was less than perfectly elastic, would your calculation in part (a) overestimate or underestimate the true excess burden? Why?

Now suppose you are given the two additional pieces of information.

- The compensated elasticity of demand for food with respect to the price, denoted as 0^{food}, is equal to -0.5.
- The uncompensated elasticity of demand for food with respect to the price, denoted as $,^{food}$, is equal to -2.0.

c. Assume that clothing and food are the only goods in the economy. If the government needs to raise money and can impose an ad-valorem tax on both goods, how should it set the ratio of the tax rates, $\frac{\tau^{CLOTHING}}{\tau^{FOOD}}$, to minimize excess burden?

d. Suppose that there are only two goods, clothing and food. Illustrate how an ad valorem tax on food can lead to excess burden even if there is no change in food consumption. Clearly mark the excess burden in your picture.

C. An application to taxation of family labor supply

Under current law, the unit of taxation is family, not the individual. Both the husband and wife are taxed on the sum of their incomes. That is, if the wife earned an extra dollar, the family pays the same additional taxes to the government as if the husband had earned the extra dollar.

$U=u(C, L_{Husband}, L_{Wife})$

A tax on earnings (W_H and W_W) creates excess burden. How should tax rates be set to minimize the burden? Assume the husband's and wife's hours of work are unrelated goods -- so that an increase in one's wage does not affect the other's hours.

The inverse elasticity rule implies imposing a higher tax rate on commodity that is relatively inelastically supplied. To enhance efficiency, whoever's labor supply is relatively inelastic should bear the relatively high tax rate. Among adults aged 25 to 54, the labor supply of men is much more inelastic than women; it is efficient to impose higher tax rates on men rather than women.

Example #2

As a policy maker, you are provided with the following information on the labor market.

- The compensated elasticity of hours of work supplied with respect to the after-tax wage rate for men, denoted as 0^{men}, is equal to 0.4.
- The uncompensated elasticity of hours of work supplied with respect to the after-tax wage rate for men, denoted as , ^{men}, is equal to 0.1.
- On average, men work 1,000 hours per year at a wage rate of $7 per hour.
- The demand curve for male labor is perfectly elastic.

a. If wages increase by 10 %, by what percentage will hours of work of men increase?
b. Calculate the excess burden arising from an 50 % ad-valorem tax on wages of

men.

Suppose that there are 3 goods in the economy: consumption goods (C), leisure of men (L^{men}), and leisure of women (L^{women}). Furthermore, assume that each good is neither a complement nor a substitute for the other two goods. Finally, suppose that you are provided with the following additional information:

- The compensated elasticity of hours of work supplied with respect to the after-tax wage rate for women, denoted as 0^{women}, is equal to 0.3.
- The uncompensated elasticity of hours of work supplied with respect to the after-tax wage rate for women, denoted as $,^{women}$, is equal to 0.2.
- On average, women work 800 hours per year at a wage rate of \$9 per hour.
- The demand curve for female labor is perfectly elastic.

c. If the government is able to impose an ad-valorem tax all three goods, what will be the ratio of the tax rate on the wages of women to the tax rate on the wages of men that will minimize excess burden, that is, $\frac{\tau^{WOMEN}}{\tau^{MEN}}$?

d. If the government is only able to impose an ad-valorem tax on the leisure of women and the leisure of men but not on consumption goods, what will be the ratio of the tax rate on the wages of women to the tax rate on the wages of men that will minimize excess burden? Do men or women face a higher tax rate?