LECTURE 9
TAXATION AND EFFICIENCY

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I. BRIEF INTRODUCTION

- Suppose the government imposes a tax on food, and the person stops buying food. Is the person unaffected by the tax, since he or she pays nothing? No ... as long as the person was initially choosing his or her utility maximizing bundle, this change in behavior leads to lower utility.

- If a tax changes economic decision making (such as reducing the consumption of food), it may bring about an excess burden -- a loss in economic welfare beyond the tax revenues raised. This welfare cost can be thought of as deadweight loss.

II. EXCESS BURDEN WITH INDIFFERENCE CURVES

- Utility maximization problem: the decision maker maximizes U(Food, Clothing) subject to income and prices.

Assume the government impose an ad-valorem tax on food, so its after-tax price per unit is $(1+\tau)P_F$.

***** SEE FIGURE 9-1 *****

- The vertical distance in this example shows how much extra clothing the person has to forego in order to consume the same amount of food as before the tax were imposed. For example, at the bundle F_2, the cost in clothing foregone is E_1E_2.
- The tax rotates the budget constraint; unless person was consuming only clothing before the tax, he or she must be worse off after it is imposed.

The key question is: what is the most efficient way to raise the revenue? If the government can raise the same amount of revenue with a smaller utility loss, then there is excess burden from the food tax illustrated above.

How much worse off is person made by moving to lower indifference curve in terms of income? The equivalent variation is the amount of income the government would have to take away from the person to induce him or her to move to lower indifference curve. Taking money away is equivalent to a parallel shift in budget line. Simply shift the budget line until it is just tangent to the lower indifference

curve.

Note that at this new point, F_3, the amount of clothing foregone (money raised) is RE_3. This vertical distance is bigger than at E_1E_2, it is instead equal to E_1S. The result: the food tax makes the person worse off in dollar terms than it raises in revenue. For example, perhaps the person is made worse off by $10, but it only raises $5. The excess burden would be the gap E_2S.

Not every tax entails excess burden, however. A lump sum tax is defined as a tax that must be paid regardless of the taxpayer's behavior. Unlike ad-valorem taxes, lump sum taxes do not change relative prices. The food tax is not a lump sum tax because the revenue raised depends on the person's food consumption.

A lump sum tax can leave the person just as well off (that is, on same indifference curve). The lump sum tax must be parallel to initial budget set because the government is simply taking away income. In addition, the new budget constraint must touch lower indifference curve. How much revenue does this lump sum tax raise? The amount RE_3. That revenue equals the equivalent variation (by definition). Since revenue raised equals the equivalent variation, a lump sum tax has no excess burden. In other words, lump sum tax generates more revenue for government than food tax, but leaves the person on the same indifference curve.

III. Excess Burden: Questions and Answers

A. If lump sum taxes are so efficient, why aren't they widely used?

- political constraints and unfairness

I.e. simply taking away $2000 per year per person could be perceived as unfair, since some people are more needy than others.

In 1990, Margaret Thatcher, the former prime minister of England, implemented a "poll tax" -- a lump sum tax per person. It was very unpopular, and there were compliance problems.

It may seem reasonable to condition the tax payment on income. If the government

tries to condition on income, however, the tax payment loses the lump sum aspect. Part of a person's income could be derived from labor supply, and leisure is a "good" in a utility maximization problem.

- To construct an equitable system of lump sum taxes, the government would need to figure out earnings potential, not realized earnings. But this is clearly impossible to implement.

B. If the demand for a commodity does not change when it is taxed, is there excess burden? Yes.

Graphically:

***** SEE FIGURE 9-2 *****

Here $F_1=F_2$ even after the ad-valorem tax on food is imposed. Note that the equivalent variation is RE_3, which is greater than the revenue raised, E_1E_2. The excess burden is the distance E_2S.

Why doesn't food consumption change? There are two different effects of increasing the price of food, the "income effect" and the "substitution effect." The income effect is the change in food demand moving from one indifference curve to another, while the substitution effect is the change in food demand moving along one indifference curve (thus, holding utility constant).

The sum of the income and substitution effects is called the uncompensated response. If food was an inferior good, the income effect implies food consumption increases after a price increase. If the income effect exactly offsets the substitution effect, there is no change in food consumption. When we derive demand curves from indifference curves, we simply change prices and see the total response to quantity. The demand curves that measure both responses are called "Marshallian demand curves."

Graphically, the movement from E_1 to E_3 is the income effect: the change in food consumption due to less income, holding relative prices constant. The movement from E_3 to E_2 is substitution effect -- the change in relative prices of goods, and

compensating the person to remain on the same indifference curve as before.

The compensated response (or substitution effect) is needed for excess burden to arise. By construction, compare tax collections at E_2 and E_3.

Marshallian ordinary demand curve: how Q changes when P changes.

Hicksian compensated demand curve: how Q changes when P changes AND simultaneously giving or taking away income so that the individual's commodity bundle remains on the same indifference curve.

Exercise on deriving demand curves -
1. Ordinary (Marshallian) demand curves - change price in (F,C) space, see what happens to food consumption. With Cobb-douglas utility function, leads to our "usual" demand function.

2. Compensated (Hicksian) demand curves - only the substitution effect, holding utility constant.

3. Algebraically, substitute the ordinary demand curves into the utility function to get the indirect utility function --> V=f(income, prices). Then rearrange, so that income is a function of indirect utility and prices. Then substitute in for income into the ordinary demand curves to get the Hicksian demand curves. Note that Hicksian demand curves are a function of all prices, even if the ordinary demand curve is not.

Example 1

From the utility function $U = F^{\frac{1}{2}}C^{\frac{1}{2}}$, the ordinary demand curves are $F = \frac{1}{2}\left(\frac{I}{P_F}\right)$ and $C = \frac{1}{2}\left(\frac{I}{P_C}\right)$.

Substitute these demand curves into the utility function to get the indirect utility function:

$$V = F^{\frac{1}{2}}C^{\frac{1}{2}} = \left(\frac{1}{2}\frac{I}{P_F}\right)^{\frac{1}{2}}\left(\frac{1}{2}\frac{I}{P_C}\right)^{\frac{1}{2}}$$

which can be rewritten as:

$$V = \left(\frac{1}{2}^{\frac{1}{2}} I^{\frac{1}{2}} P_F^{-\frac{1}{2}} \right) \left(\frac{1}{2}^{\frac{1}{2}} I^{\frac{1}{2}} P_C^{-\frac{1}{2}} \right)$$

and then, by rearranging the terms,

$$V = \frac{1}{2}^{\frac{1}{2}} \frac{1}{2}^{\frac{1}{2}} I^{\frac{1}{2}} I^{\frac{1}{2}} P_F^{-\frac{1}{2}} P_C^{-\frac{1}{2}}$$

which then simplifies to:

$$V = \frac{I}{2 P_F^{\frac{1}{2}} P_C^{\frac{1}{2}}}$$

We can then rearrange the above equation in terms of income:

$$I = 2V P_F^{\frac{1}{2}} P_C^{\frac{1}{2}}$$

And then substitute this expression in for income into the ordinary demand curves:

$$F = \frac{1}{2} \left(\frac{I}{P_F} \right)$$

$$F = \frac{1}{2} \frac{\left(2V P_F^{\frac{1}{2}} P_C^{\frac{1}{2}} \right)}{P_F}$$

which then simplifies to:

$$F = V \left(\frac{P_C}{P_F} \right)^{\frac{1}{2}}$$

and by similar calculations,

$$C = \frac{1}{2} \frac{\left(2V P_F^{\frac{1}{2}} P_C^{\frac{1}{2}} \right)}{P_C}$$

which simplifies to:

$$C = V \left(\frac{P_F}{P_C} \right)^{\frac{1}{2}}$$

4. Recall that the Slutsky equation breaks down the income and substitution effects -- the income effect is moving from one indifference curve to another, holding

prices constant, while the substitution effect is moving along an indifference curve, hold utility constant.

5. Slutsky equation: $\frac{dQ}{dP} = \frac{dH}{dP} - Q\frac{dQ}{dI}$, where $\frac{dQ}{dP}$ is the ordinary (Marshallian) demand, $\frac{dH}{dP}$ is the change in Hicksian demand with respect to price, Q=quantity, and $\frac{dQ}{dI}$ is the change with respect to income.

6. Can rearrange Slutsky equation in terms of elasticities: multiply each term in the above equation by $\frac{P}{Q}$. The first two terms turn into price elasticities. The final term is messy. Multiply the final term by one, in the form of $\left(\frac{I}{Q}\right)\left(\frac{Q}{I}\right)$. Then the final term contains the income elasticity, and the rest of the expression simplifies into the share of the budget spent on that good.

uncompensated price elasticity = compensated price elasticity - (share of budget*income elasticity)

$$\varepsilon_P = \varepsilon_P^H - s\varepsilon_I$$

Recall that elasticities are $\frac{\%\Delta DependentVariable}{\%\Delta IndependentVariable}$

Example 2
If the uncompensated demand elasticity $\varepsilon_P = -0.50$, and the compensated elasticity was $\varepsilon_P^H = -0.20$, then $s\varepsilon_I = 0.30$ (note the negative sign in the Slutsky equation). The term s has to be positive, which implies that E_I is positive – the good in question is a normal good.

IV. Excess Burden Measurement with Demand Curves

A. Measurement
Consider downward sloping demand curve and perfectly elastic supply. Recall the

notion of consumer surplus: how much are consumers willing to pay versus how much they actually pay for the good.

***** SEE FIGURE 9-3 *****

Suppose tax again levied on food: price is now $(1+_J)P_F$. In this case the supply curve shifts upward.

In this case:
1. Consumers lose the area between the two supply curves.
2. Revenue is equal to the rectangle, the number of units purchased times tax price per unit.
3. The sum of the new consumer surplus plus the tax revenue is smaller than original consumer surplus.

Dollar measure for excess burden (DWL):

- We can recompute the deadweight loss triangle. If we rewrite this, however, we gain some additional intuition.

(1) $$DWL = \frac{1}{2} base * height$$

(2) $$DWL = \frac{1}{2}(\Delta Q)(\Delta P)$$

Examine the term, ΔP. It is just difference in net and gross prices :

(3) $$\Delta P = (1+\tau)P - P = \tau P$$

Now examine the term ΔQ, and recall the definition of a compensated demand elasticity:

(4) $$\eta = \frac{\Delta Q}{\Delta P}\frac{P}{Q}$$

Rewrite (4) in terms of ΔQ:

(5) $$\Delta Q = \eta\left(\frac{Q}{P}\right)\Delta P$$

From the definition of ΔP in equation (2):

(6) $$\Delta Q = \eta\left(\frac{Q}{P}\right)\tau P$$

Which simplifies to:

(7) $$\Delta Q = \eta Q \tau$$

Then substitute the definitions of (3) and (7) into formula (2):

(8) $$DWL = \frac{1}{2}\eta Q \tau \tau P$$

-- or --

(9) $$DWL = \frac{1}{2}|\eta|(PQ)\tau^2$$

where $|\eta|$ is the absolute value of the compensated price elasticity of demand.

WHY GO THROUGH THIS CALCULATION? The following implications emerge.

1. High $|\eta|$ implies high DWL.
   - 0 means big change in Q to change in P. Implies big distortion in decision.
2. PQ = total revenue spent on food initially. Greater share spent initially greater the excess burden.
3. J^2 -- as tax rate increases, excess burden goes up with its square!
   - Everything else equal, doubling a tax quadruples its excess burden.
   - The marginal excess burden greater than average.

B. Preexisting Distortions

- monopoly
- externalities
- preexisting taxes

The analysis is complicated when there are already preexisting distortions. It is not necessarily true that two distortions are worse than one.

For example: recall the tax on food, J. This tax changed relative prices. If the government then imposed a tax on the other good, clothing to return to the original slope of the budget set, there is less excess burden than before.

C. The excess burden of a subsidy

Just as a tax imposes an excess burden, so does a subsidy. Graphically, suppose that the government gives a subsidy of s % to housing producers.

***** SEE FIGURE 9-4 *****

Consumer surplus has increased, but from society's point of view, the subsidy is not worthwhile because the extra people that buy housing value it at less than its marginal cost.

Important policy implication: help some group by subsidizing a commodity that they use heavily. Inefficient way to aid people. Better to give as direct grant. Graphically, the group above would be indifferent to the smaller area that is the change in the consumer surplus to the larger area which is the cost to the government.

D. The excess burden of income taxation

Graphically, upward sloping labor supply curve:

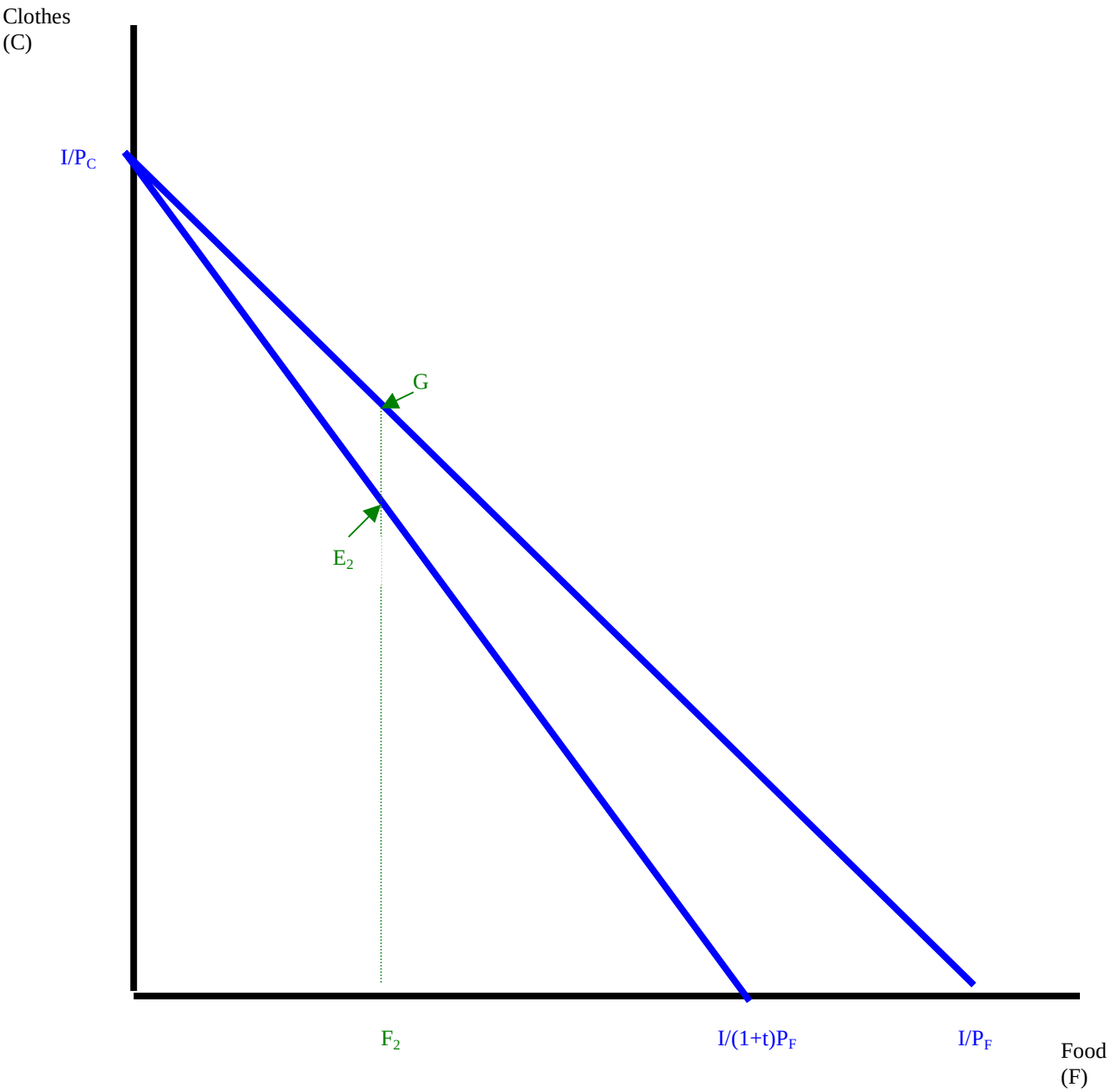
***** SEE FIGURE 9-5 *****

Recall, the formula for the excess burden is:

½ , wL_1J^2

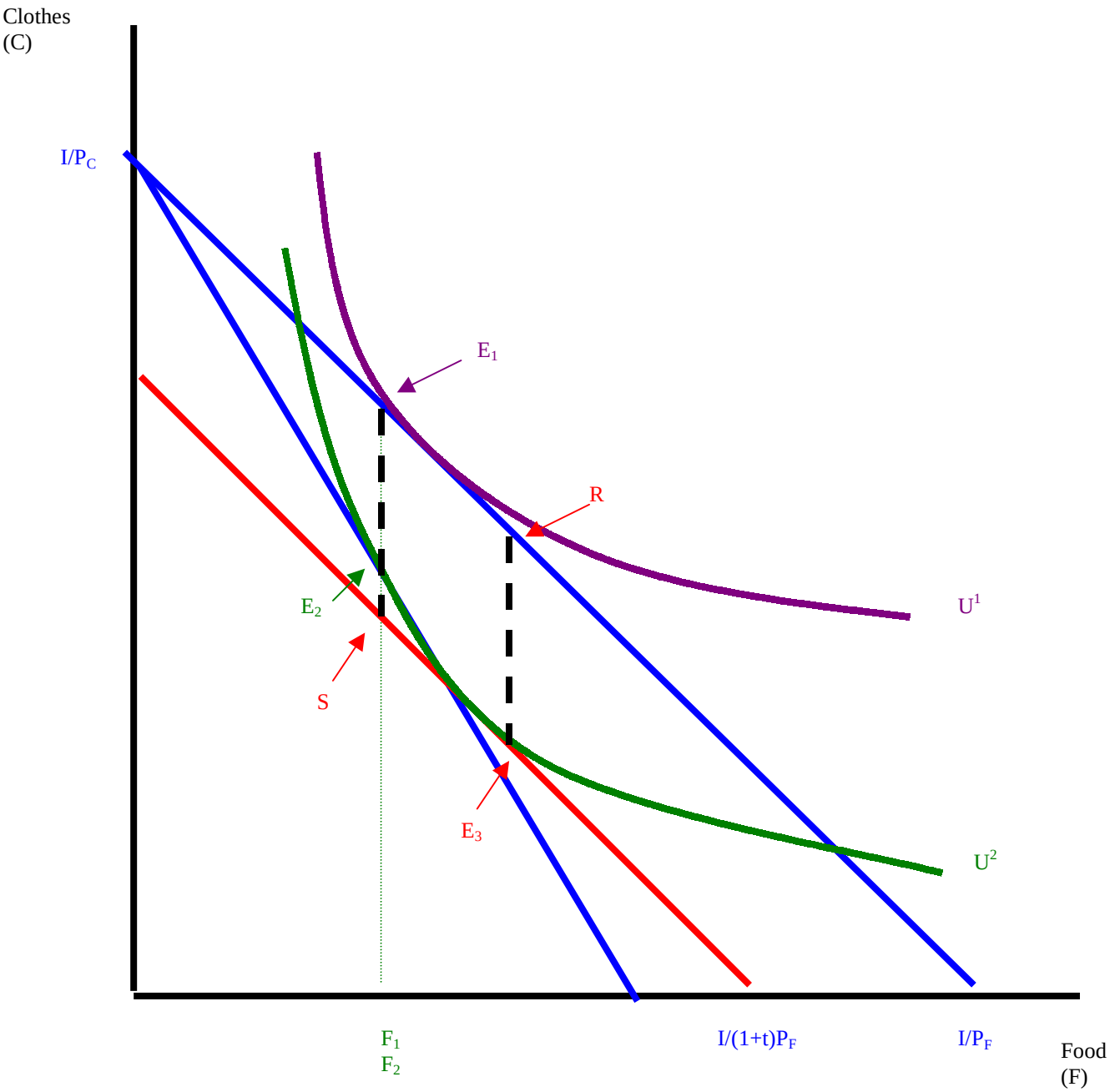
where g is the elasticity of hours of work with respect to the wage. Let $\varepsilon = 0.2, L_1 = 2000, w = 20, \tau = 0.4$. Then excess burden is $640 annually. Tax revenue $= \tau w L_1 = 16000$. Thus $\frac{640}{16000} = 4\%$ of tax revenue.

Figure 9-1 – Ad valorem tax on food



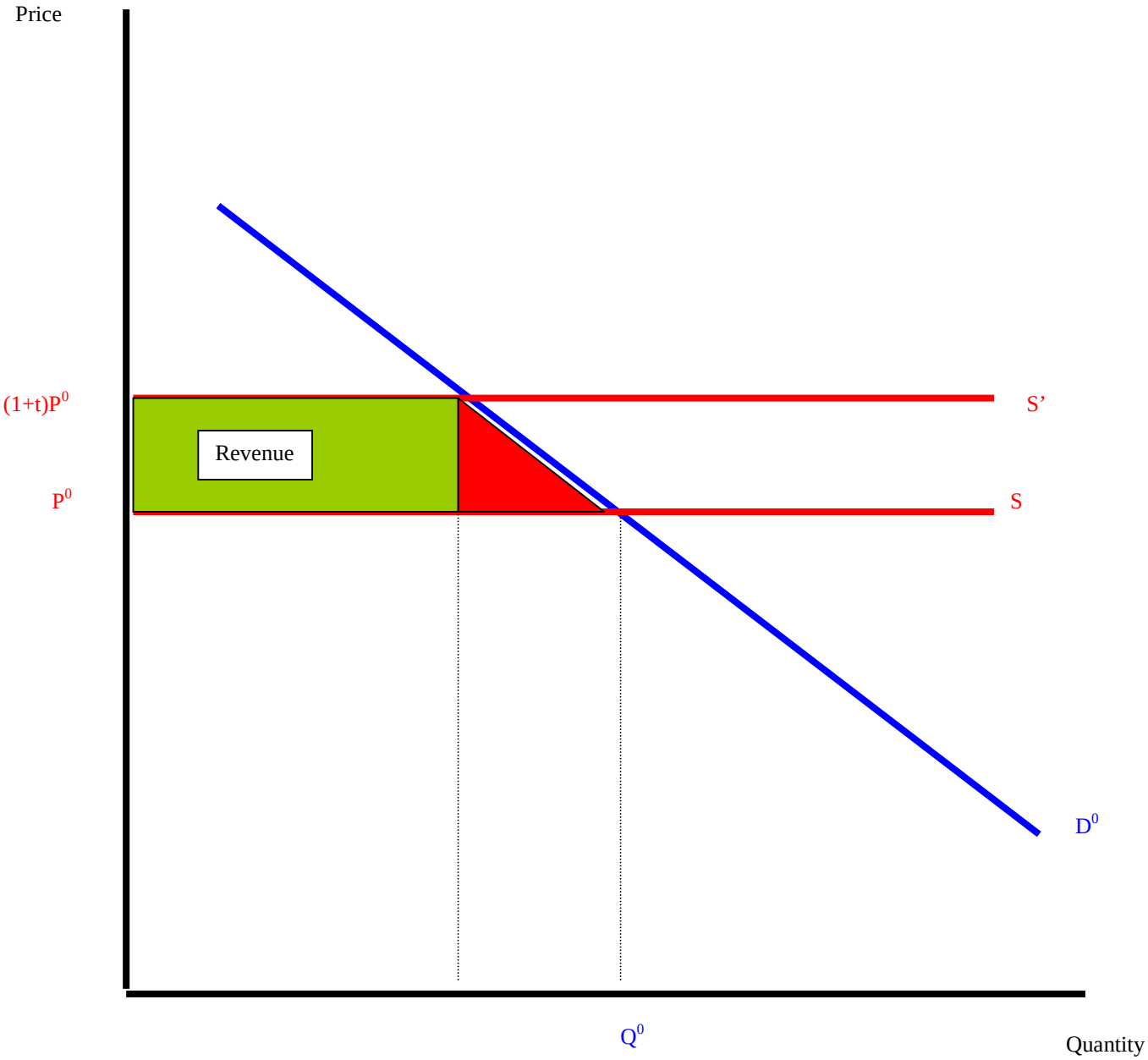
Vertical distance GE_2 is the amount of Clothes foregone if food constant remains at F_2.

Figure 9-2 – Deriving excess burden graphically



Both E_2 and E_3 give the same utility. But government raises more money at E_3, as shown by the dashed black line.

Figure 9-3 – Measuring excess burden with Hicksian demand curves

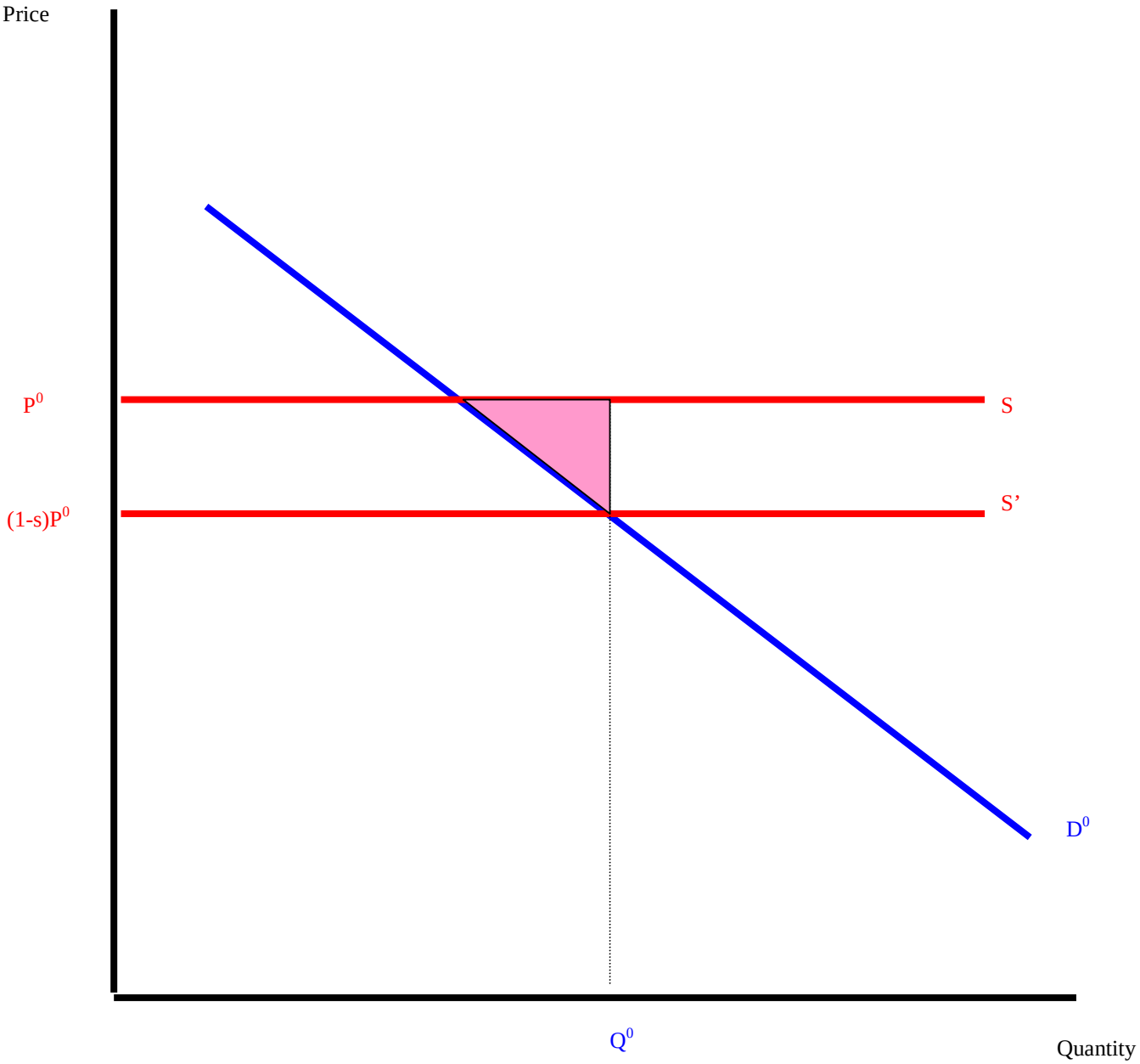


To compute excess burden we assume:

(a) The supply curve is perfectly elastic, and

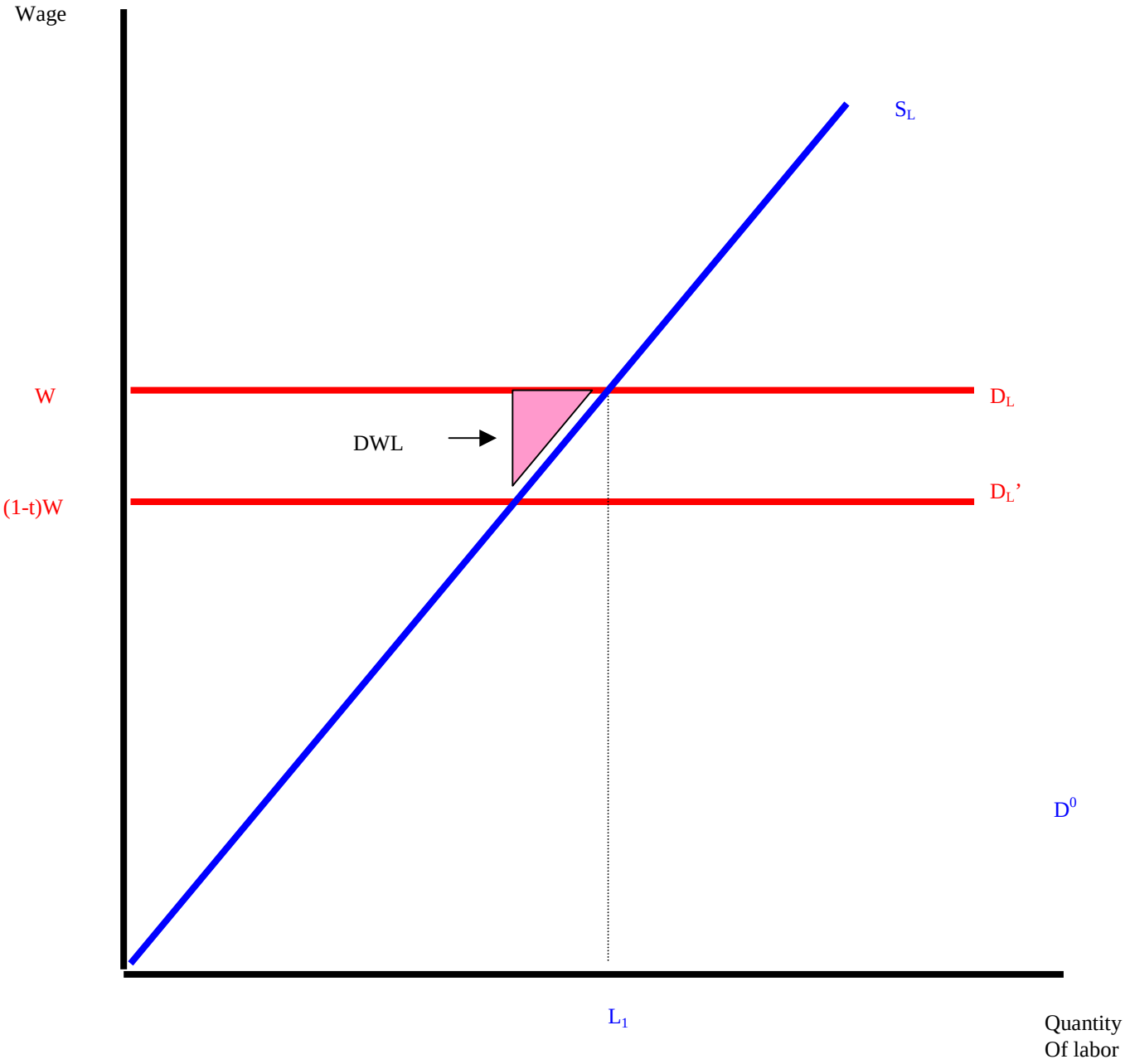
(b) The downward sloping demand curve is the Hicksian compensated demand curve.

Figure 9-4 – Measuring excess burden with a subsidy



With a subsidy, too much of the good is produced and there is deadweight loss.

Figure 9-5 – Excess burden from income taxation



Given a wage, compensated supply elasticity, initial hours of work, and tax rate, we can compute the excess burden of the tax.