LECTURE 1

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Relevant readings: Rosen textbook, Chapter 1 and Appendix, “Some basic microeconomics,” pp. 507-527.

I. GOALS OF PUBLIC FINANCE AND THIS COURSE

A. Public finance is knowing what activities the public sector engages in and how they are organized. A lot of people might have misconceptions about what the programs are out there and why they really are important. Some examples from back when I was a student:

1. "If I quit my job or get fired, I can just go onto welfare or collect unemployment insurance."
2. "Why is everyone so concerned about health care? I'm healthy, why should I care? What is an HMO and how do they control costs?"
3. "What is an IRA? Why do people seem to talk about them right a lot right around April 15th when taxes are due?"
4. "What is a marginal tax rate? I know that the tax system has different rates, i.e. 15%, 28%, etc. What does that mean for my take home pay?"
5. "The cost of welfare programs is driving the debate behind welfare reform."

B. Understanding and anticipating the full consequences of governmental activities.
1. Who wins and loses, different behavioral responses.
2. "Rent control benefits the poor."

C. Evaluating alternative policies.
1. To get families off of welfare: cut back benefits vs. job training. Short vs. long run effects.

II. HOW DOES THE GOVERNMENT SPEND THE MONEY IT RAISES?

A. Some of the more important items include national defense, social security, public welfare, and interest on the debt.

B. Total expenditure in 1991 by government was 2.4 trillion. Who spends it?
1. Federal government (defense, social security, other stuff): 55%
2. States (welfare, education): 19%
3. Localities (police, fire protection): 26%

III. REVIEW OF BASIC MICROECONOMICS

A. Utility Function: Preferences over different goods

For example, U=u(Food, Clothing). This can be generalized to more than two goods, and in some case we will do that. Consider the following budget constraints in (F,C) space.

***** SEE FIGURE 1-1 *****

1. Non-satiation assumption = more is preferred to less; A provides more than C
2. Transitivity = if $A \succ B$ and $B \succ C$, then $A \succ C$.
3. Revealed Preference = If point B is chosen, it is (weakly) preferred to all other points on that budget line.
4. Derive Indifference Curves: all the (F,C) bundles that keep utility constant.

B. Monotonic Transformations

Consider the Cobb-Douglas utility function: $u = C^{\frac{1}{3}}F^{\frac{2}{3}}$.
Another utility function that preserves these preferences is:

$$\tilde{u} = \log u = \log\left(C^{\frac{1}{3}}F^{\frac{2}{3}}\right) = \frac{1}{3}\log C + \frac{2}{3}\log F$$

Yet another utility function that preserves these preferences is

$$\hat{u} = 20u = 20C^{\frac{1}{3}}F^{\frac{2}{3}}$$

Note: In all cases, the shape of the indifference curve remains the same, the only thing that changes is the utility associated with it. Why do this? In some cases, transforming the utility function makes the problem easier to solve when differentiating.

C. Marginal Rate of Substitution (MRS): The tradeoff between two goods, holding utility constant = the slope of the indifference curve.

You should know how to Derive the MRS (and then demand curve):

Marginal rate of substitution for the utility function $u = C^{\frac{1}{3}} F^{\frac{2}{3}}$

In this case, the MRS equals the ratio of marginal utilities:

$$MRS = -\frac{dC}{dF} = \frac{MU_F}{MU_C} = \left(\frac{2}{3} C^{\frac{1}{3}} F^{-\frac{1}{3}} \Big/ \frac{1}{3} C^{-\frac{2}{3}} F^{\frac{2}{3}} \right) = 2\frac{C}{F}$$

Note: If took logarithms instead (monotonic transformation) would get:

$$\tilde{u} = \frac{1}{3}\log C + \frac{2}{3}\log F$$

$$MRS = \left(\frac{2}{3}\frac{1}{F} \Big/ \frac{1}{3}\frac{1}{C} \right) = 2\frac{C}{F}$$

, as before.

Let P_F and P_C be the prices of food and clothing.
Then set MRS=ratio of prices

$$2\frac{C}{F} = \frac{P_F}{P_C}$$

or $2P_C C = P_F F$.

And from the budget constraint, $P_C C + P_F F = I$

Substituting this in, we get $3P_C C = I$, or $C^* = \frac{1}{3}\frac{I}{P_C}$.

For the Cobb Douglas, we spend a constant share of our income on food, a constant share on clothing.

D. *When setting MRS= ratio of prices, how do we know which of the prices is in the numerator?*

Go through the ”Bang per Buck” argument

$$\frac{MU_F}{P_F} = \frac{MU_C}{P_C}$$ when maximizing utility

Otherwise: if $\frac{MU_F}{P_F} > \frac{MU_C}{P_C}$, what should the consumer do?

Rearrange to get: $\frac{MU_F}{MU_C} = \frac{P_F}{P_C}$, or $MRS = \frac{P_F}{P_C}$

E. KEY UTILITY FUNCTIONS

1. Perfect complements:

$u(F,C) = \min\{\alpha F, \beta C\}$ ==> choose (F,C) such that $\alpha F = \beta C$.

Leads to "L" shaped indifference curves.
For example, left-shoes and right-shoes.

$$P_C C + P_F F = I$$

$$\alpha F = \beta C \Rightarrow F = \frac{\beta}{\alpha} C$$

$$P_C C + P_F \frac{\beta}{\alpha} C = I$$

$$C^* = \frac{I}{P_C + P_F \frac{\beta}{\alpha}}$$

$$F^* = \frac{I}{P_F + P_C \frac{\alpha}{\beta}}$$

Note: Food and Clothing are not good examples of perfect complements.
Note 2: Can't take derivatives of min {...} function.

2. Perfect substitutes:

$u(F,C) = \alpha F + \beta C$ ==> constant MRS: $\frac{MU_F}{MU_C} = \frac{\alpha}{\beta}$

- The indifference curves are straight lines. Unless relative

prices are also equal to $\frac{\alpha}{\beta}$, will end up at a corner solution.
- To solve the demand curve, note that as long as MRS=ratio of prices, all bundles on budget constraint give same utility.

3. Cobb-Douglas Utility:

Consider the utility function $u(F,C) = F^{\alpha}C^{\beta}$ which has equivalent preferences as $\tilde{u} = F^{\frac{\alpha}{\alpha+\beta}}C^{\frac{\beta}{\alpha+\beta}}$.

The way to see that these are the same is to note that $\tilde{u} = u^{\frac{1}{\alpha+\beta}}$, and that is a monotonic transformation.

-That is, $u = F^2 C$ gives the same indifference curves as $\tilde{u} = F^{\frac{2}{3}}C^{\frac{1}{3}}$. Put differently, $\tilde{u} = u^{\frac{1}{3}}$.

-To solve, one could transform utility function by taking logs, e.g., by going back to the first expression,

$$\bar{u} = \log \tilde{u} = \log\left(F^{\frac{\alpha}{\alpha+\beta}}C^{\frac{\beta}{\alpha+\beta}}\right) = \log F^{\frac{\alpha}{\alpha+\beta}} + \log C^{\frac{\beta}{\alpha+\beta}} = \frac{\alpha}{\alpha+\beta}\log F + \frac{\beta}{\alpha+\beta}\log C$$

- Then set $\left(\frac{\partial \bar{u}}{\partial F}\Big/P_F\right) = \left(\frac{\partial \bar{u}}{\partial C}\Big/P_C\right)$, which is the expression for setting the marginal rate of substitution equal to the ratio of prices. This is the first condition we need for utility maximization. We first find the marginal utility of food:

$$\frac{\partial \bar{u}}{\partial F} = \frac{\partial\left(\frac{\alpha}{\alpha+\beta}\log F + \frac{\beta}{\alpha+\beta}\log C\right)}{\partial F} = \frac{\alpha}{\alpha+\beta}\frac{1}{F}$$

, because if you recall from

calculus, the partial derivative $\frac{\partial \log F}{\partial F} = \frac{1}{F}$, and the partial derivative of the second part of the expression, $\frac{\beta}{\alpha+\beta}\log C$, with respect to F is zero because it is treated as a constant term. Then dividing the marginal utility of food by its price gives us $\left(\frac{\partial u}{\partial F}\Big/ P_F\right) = \frac{\alpha}{\alpha+\beta}\frac{1}{F}\Big/ P_F = \frac{\alpha}{\alpha+\beta}\frac{1}{P_F F}$.

- If you did a similar exercise for clothing, you would get $\left(\frac{\partial u}{\partial C}\Big/ P_C\right) = \frac{\beta}{\alpha+\beta}\frac{1}{C}\Big/ P_C = \frac{\beta}{\alpha+\beta}\frac{1}{P_C C}$.

- Therefore, setting the marginal utilities divided by their prices leads to: $\frac{\alpha}{\alpha+\beta}\frac{1}{P_F F} = \frac{\beta}{\alpha+\beta}\frac{1}{P_C C}$, which simplifies to $\alpha P_C C = \beta P_F F$ (once we multiply both sides of the equation by $(\alpha+\beta)$ and cross multiply).

- We can then express expenditure on food, $P_F F$, in terms of the other variables so that we can substitute this into the budget constraint. Rewriting the equation, expenditure on food is: $P_F F = \frac{\alpha}{\beta} P_C C$.

- We then substitute this into the budget constraint, which is given by $P_C C + P_F F = I$. Thus, $P_C C + \frac{\alpha}{\beta} P_C C = I$.

- This gives us an expression with one unknown variable, the quantity of clothing (C). We can rewrite this last equation as $\frac{\beta}{\beta} P_C C + \frac{\alpha}{\beta} P_C C = I$, which in turn can be expressed as $\frac{\beta+\alpha}{\beta} P_C C = I$,

or $\frac{\beta+\alpha}{\beta} C = \frac{I}{P_C}$, or finally, $C^{*} = \frac{\beta}{\alpha + \beta}\frac{I}{P_C}$.

- Through similar steps, we would find that $F^{*} = \frac{\alpha}{\alpha + \beta}\frac{I}{P_F}$.

- We could also solve for the initial utility function $u(F, C) = F^{\alpha} C^{\beta}$. First set the MRS equal to the ratio of the prices. Computing the marginal utilities, we have $\frac{\partial u}{\partial F} = \alpha F^{\alpha-1} C^{\beta}$ and $\frac{\partial u}{\partial C} = \beta F^{\alpha} C^{\beta-1}$. Therefore,

$$\frac{MU_F}{MU_C} = \frac{\frac{\partial u}{\partial F}}{\frac{\partial u}{\partial C}} = \frac{\alpha F^{\alpha-1} C^{\beta}}{\beta F^{\alpha} C^{\beta-1}} = \frac{\alpha F^{\alpha-1} C^{\beta}}{\beta F^{\alpha} C^{\beta-1}}\left(\frac{F^{1-\alpha}}{F^{1-\alpha}}\frac{C^{1-\beta}}{C^{1-\beta}}\right) = \frac{\alpha C}{\beta F} = \frac{P_F}{P_C}$$

, which simplifies to $\alpha P_C C = \beta P_F F \Rightarrow P_F F = \frac{\alpha}{\beta} P_C C$. If we substitute into the budget constraint, given by $P_C C + P_F F = I$, for food expenditure ($P_F F$), we get $P_C C + \frac{\alpha}{\beta} P_C C = I$ which can be rewritten as $\frac{\beta}{\beta} P_C C + \frac{\alpha}{\beta} P_C C = I$, or $\frac{\alpha + \beta}{\beta} P_C C = I$, which then can be rewritten as the demand curve for clothing, $C^{*} = \frac{\beta}{\alpha + \beta}\frac{I}{P_C}$.

- Suppose you were given the following utility function: $\tilde{u} = F^{\frac{2}{3}} C^{\frac{1}{3}}$.

Then $\alpha = \frac{2}{3}$ and $\beta = \frac{1}{3}$. If we substitute these into the demand equations for F^{*}, we would get $F^{*} = \left(\frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{3}}\right)\frac{I}{P_F}$, or $F^{*} = \frac{2}{3}\frac{I}{P_F}$. A similar calculation for C^{*} would give $C^{*} = \frac{1}{3}\frac{I}{P_C}$. If you were given prices and income, you could compute the exact amount of food and clothing that the consumer would choose.

- As an exercise, you should be able to derive the demand curves for the utility function $\tilde{u} = F^{\frac{4}{5}}C^{\frac{5}{4}}$. As a hint, the demand curve for food is $F^* = \left(\frac{16}{41}\right)\frac{I}{P_F}$. Try to figure out the demand curve for clothing on your own.